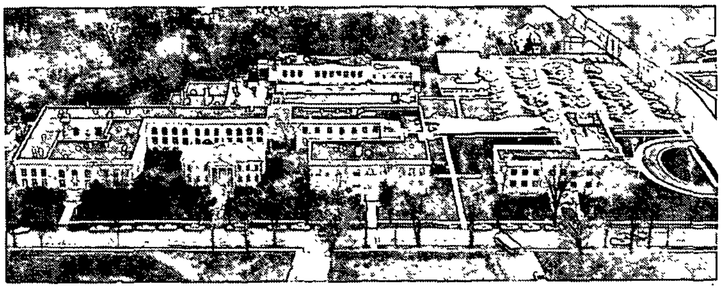

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THE ELASTIC PROPERTIES OF PAPER: A REVIEW

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The Elastic Properties of Paper: A Review

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Abstract

Paper is an interesting engineering material in that its properties can be varied over extremely broad ranges by changing machine and process variables. The elastic properties, in particular, are very sensitive indicators of such process changes. The three dimensional elastic response of paper has not received much attention in the past because of difficulties in measuring most of the relevant parameters. These parameters, involving both in-plane and out-of-plane directions, can now be measured using sound wave propagation techniques. The results to date suggest that these elastic properties of paper are significant factors in understanding the collective effects of machine and process changes and perhaps more importantly, can be used in many instances to predict end use performance of the product.

This review (1) discusses paper as a three dimensional orthotropic elastic material, (2) shows how certain paper machine variables affect elastic properties, (3) reviews the various network and continuum models developed over the years to relate sheet properties and fiber properties, (4) briefly discusses the viscoelastic behavior of paper, and (5) describes how elastic parameters are capable of predicting end use performance in many cases. Areas where our understanding is particularly weak, or where new or additional work is needed, are pointed out.

Orthotropic Behavior

Paper can be considered an orthotropic elastic material (1,2). An orthotropic material is one which has three mutually perpendicular planes of symmetry. For such a material the stresses, ε_{ij}, can be expressed in terms of the strain, τ_{ij} by

$$
\begin{aligned}
\tau_{11} &= C_{11}\varepsilon_{11} + C_{12}\varepsilon_{22} + C_{13}\varepsilon_{33} \\
\tau_{22} &= C_{12}\varepsilon_{11} + C_{22}\varepsilon_{22} + C_{23}\varepsilon_{33} \\
\tau_{33} &= C_{13}\varepsilon_{11} + C_{23}\varepsilon_{22} + C_{33}\varepsilon_{33} \\
\tau_{23} &= 2C_{44}\varepsilon_{23} \\
\tau_{13} &= 2C_{55}\varepsilon_{13} \\
\tau_{12} &= 2C_{66}\varepsilon_{12}
\end{aligned}
$$

The nine C_{ij} are called the elastic stiffnesses and have units of stress (Pa). Alternatively the nine stiffnesses can be written in terms of elastic compliances, S_{ij}, where $[S_{ij}][C_{ij}]=I$, or as engineering elastic constants. The latter include three Young's moduli, three shear moduli, and three Poisson ratios. While the elastic behavior of paper can be expressed in any of these three forms, the elastic stiffnesses are preferred since these may be measured directly using sound wave propagation techniques (3-5). Such techniques are valid as long as the wavelength of the sound wave is long compared to the characteristic dimensions of the fibers. In such cases the paper may be considered a homogeneous continuum.

The meaning of these elastic parameters can be understood by referring to Figs. 1 to 3. Figure 1 defines the three principal directions. The machine direction is referred to as MD (or x or 1), the cross direction as CD (or y or 2), and the thickness direction as ZD (or z or 3). If we apply a uniaxial stress to the sample in one of these three directions we could deform the small element as shown in Fig. 2. The ratio of the applied stress to the resultant strain (at small strains) is defined as the elastic modulus or Young's modulus in the direction of straining. The three modes of deformation shown thus result in three Young's moduli: E_{md}, E_{cd}, and E_{zd}. In addition, for any of the three modes shown in Fig. 2, the Poisson ratio would be defined as the ratio of the lateral contraction to the axial extension in the direction of straining. For the upper left hand figure, for example, two Poisson ratios could be defined since the specimen contracts in both the CD and ZD. These would be referred to as $U_{cd\text{-}md}$ and $U_{zd\text{-}md}$, or U_{yx} and U_{zx}, respectively. The three modes of deformation shown in Fig. 2 thus yield six Poisson ratios, but only three of these are independent. These are normally taken to be U_{xy}, U_{xz}, and U_{yz}. Figure 3 shows the three allowed modes of deformation in shear, when the applied stresses are parallel to one of the principal directions. In these cases, a push or pull on opposite sides (or faces) of the specimen results in a shear deformation as shown. The three independent shear stiffnesses shown correspond to each of the three planes of symmetry. The elastic parameters E_{md}, E_{cd}, G_{xy} ($G_{md\text{-}cd}$), and U_{xy} ($U_{md\text{-}cd}$) are referred to as in-plane elastic constants because they are all defined in the MD-CD plane. The parameters E_{zd}, G_{xz}, G_{yz}, U_{xz}, and U_{yz} are called out-of-plane elastic constants because they all involve the Z-direction. As will be discussed below, these quantities are not "constant" at all but are very sensitive to process conditions.

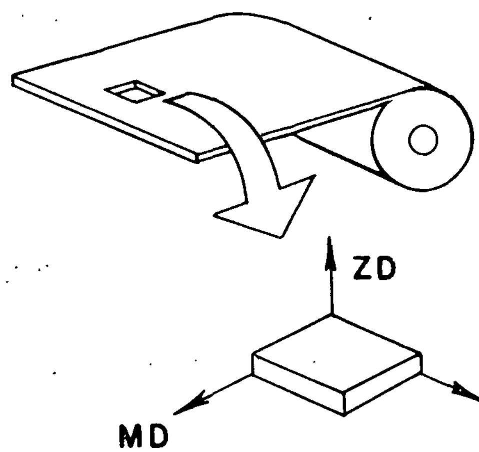

Figure 1. Principal directions assigned to paper.

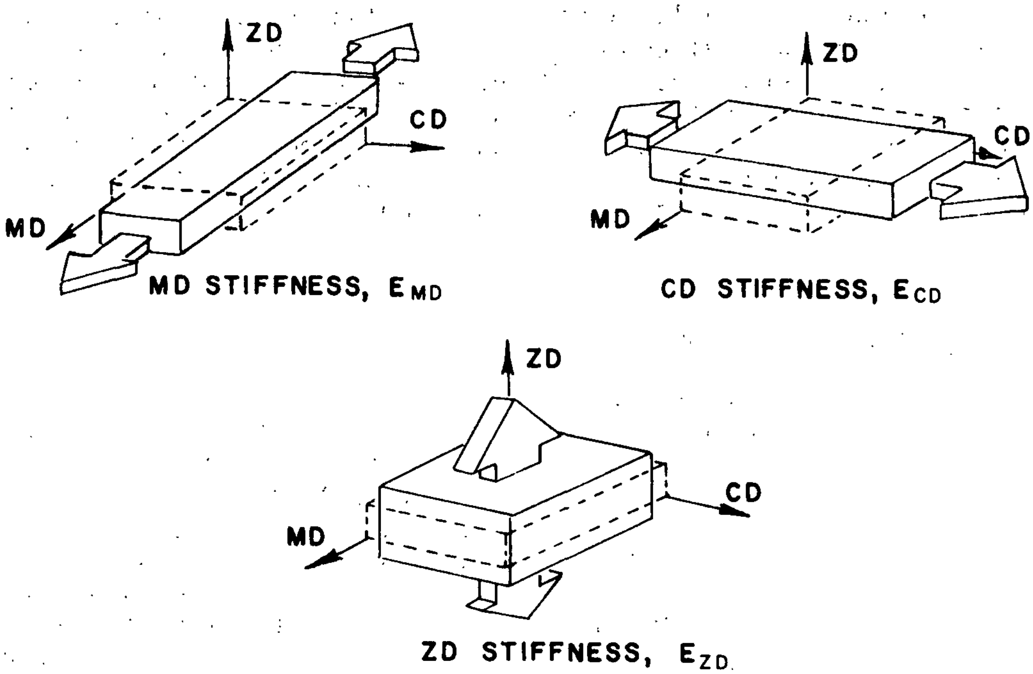

Figure 2. Three modes of deformation in uniaxial tension.

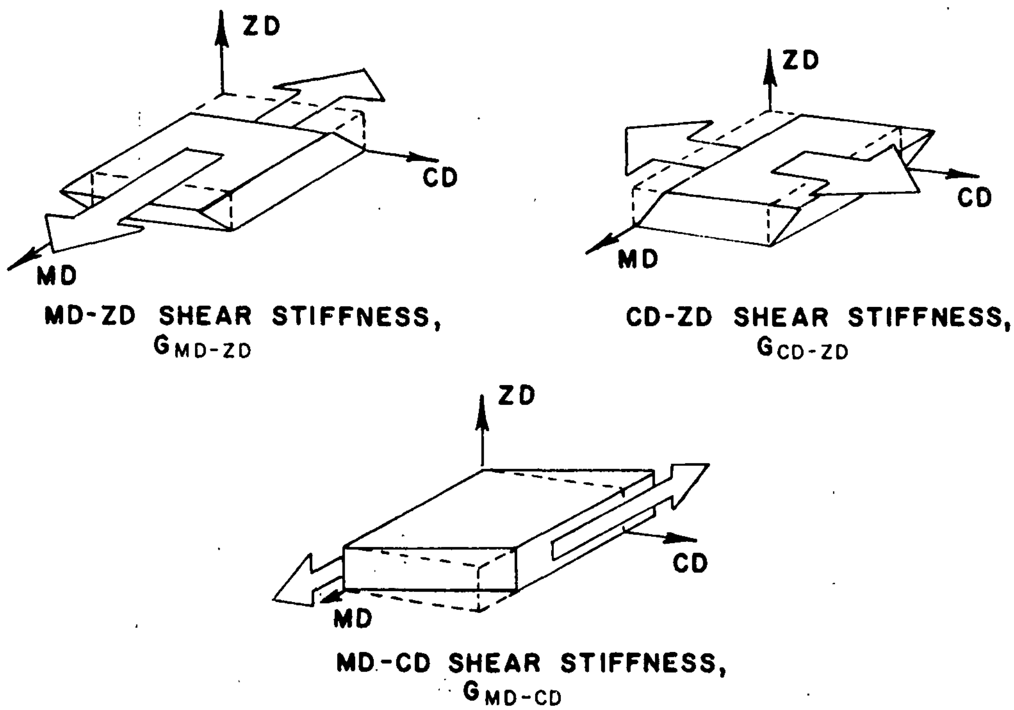

Figure 3. Three modes of deformation in shear.

The nine elastic parameters defined above are needed to completely characterize an orthotropic material. Table I gives elastic stiffnesses and engineering constants for several commercial paperboards and laboratory handsheets. For the machine made papers the MD modulus (the direction of manufacture) is two to three times greater than the CD modulus, and 150 to 200 times larger than the out-of-plane modulus, E_Z. These differences occur because of the nature of the constituent fibers and the particular events which occur on the paper machine during consolidation and drying of the paper web. These are discussed in the next section.

Elastic Properties and Machine Variables

The relationships between paper machine variables and the in-plane (MD-CD) elastic properties have been studied by a number of authors (6-16). Relationships between process variables and both in-plane and out-of-plane parameters have received less attention. Figures 4-6 illustrate how the three Young's moduli depend on fiber orientation, wet pressing (density), and wet straining (17) for a bleached softwood commercial kraft pulp refined to about 500 CSF. The fiber orientation was varied by changing the relative speeds of the pulp slurry and wire in a Formette Dynamique anisotropic sheet former and the density was changed by wet pressing. After wet pressing the sheets were strained in the MD while wet to levels of 1.2 and 2.4%. The sheets were then restrained in both the MD and CD (but not the ZD) during drying. In general, the elastic stiffness in the direction of fiber orientation and the direction of wet straining increases, while the properties in both the CD and ZD tend to decrease. Similar results are obtained for the tensile strengths in these three directions or the in-plane compressive strengths (17).

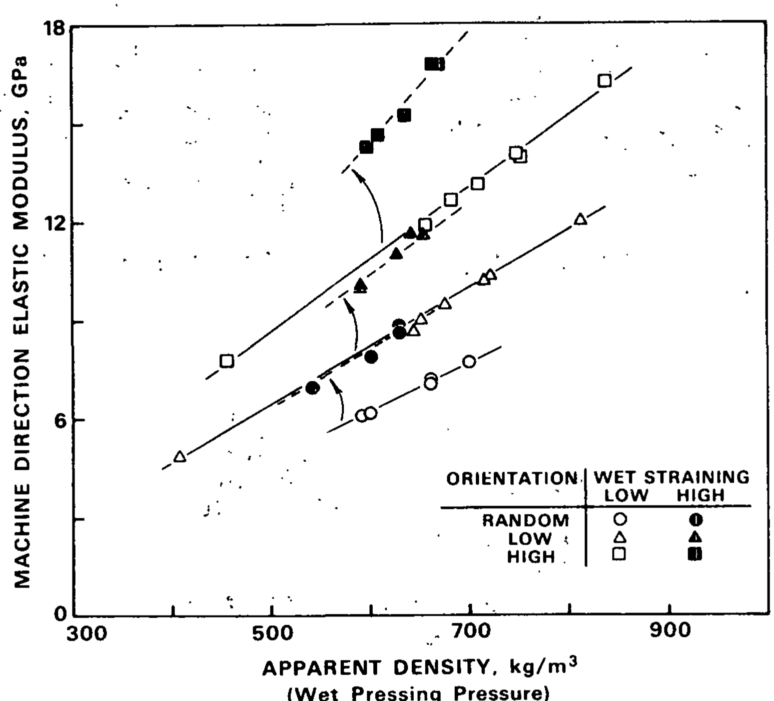

Figure 4. E_{md} vs. density with changing fiber orientation and wet straining.

TABLE I

ELASTIC PROPERTIES

	Apparent Density, ρ, kg/m^3	Stiffness[a] (GPa)									Engineering constants[b] (GPa)								
		C_{11}	C_{22}	C_{33}	C_{12}	C_{13}	C_{23}	C_{44}	C_{55}	C_{66}	E_x	E_y	E_z	ν_{xy}	ν_{xz}	ν_{yz}	G_{yz}	G_{xz}	G_{xy}
Carton stock	780	8.01	3.84	0.042	1.36	0.092	0.91	0.099	0.137	2.04	7.44	3.47	0.040	0.15	0.008	0.021	0.099	0.137	2.04
Linerboard 42 lb	752			0.059				0.050	0.060	2.08	9.98	3.39					0.050	0.060	2.08
Linerboard 90 lb	691	8.12	3.32	0.032	1.19	0.113	0.082	0.104	0.129	1.80	7.46	3.01	0.029	0.117	0.109	0.021	0.104	0.129	1.80
Boxboard	775			0.043				0.083	0.099	1.36	6.03	2.32		0.119			0.083	0.099	1.36
Laboratory BKSW[c]	721	10.9	6.40	0.172				0.290	0.343	3.09	10.3	6.04		0.182			0.290	0.343	2.97
Laboratory BKSW[c]	673	16.6	2.78	0.073				0.151	0.260	2.28	16.5	2.76		0.036			0.151	0.260	2.40
Corrugating medium	682			0.103				0.046	0.053	1.58	6.89	2.68		0.167			0.046	0.053	1.58
Linerboard 42 lb	721										8.72	4.14		0.138					2.43
Linerboard 42 lb[d]	721										6.83	3.17		0.125					1.22

[a]Three dimensional bulk stiffness.
[b]Poisson ratios are dimensionless.
[c]BKSW, bleached kraft softwood.
[d]Mechanical measurements.

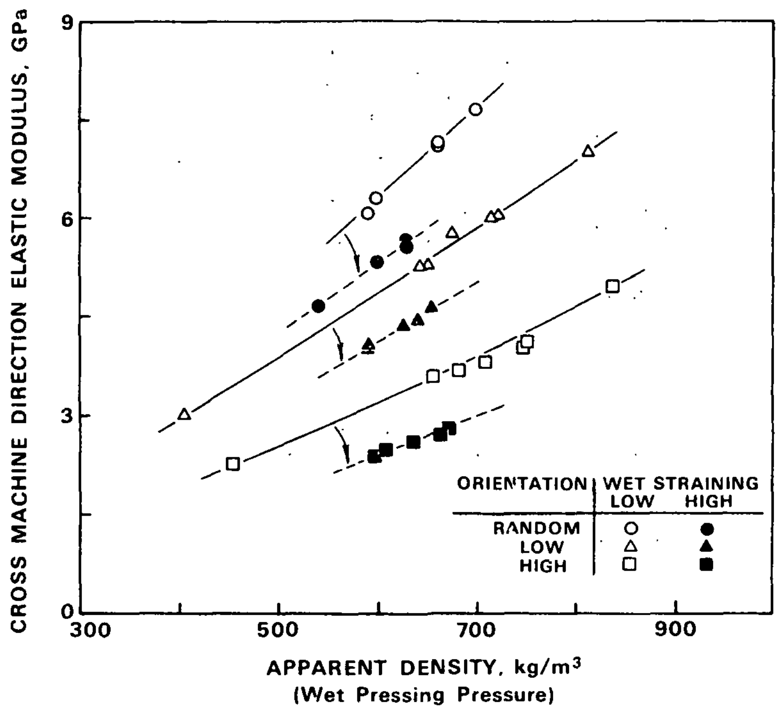

Figure 5. E_{cd} vs. density with changing fiber orientation and wet straining.

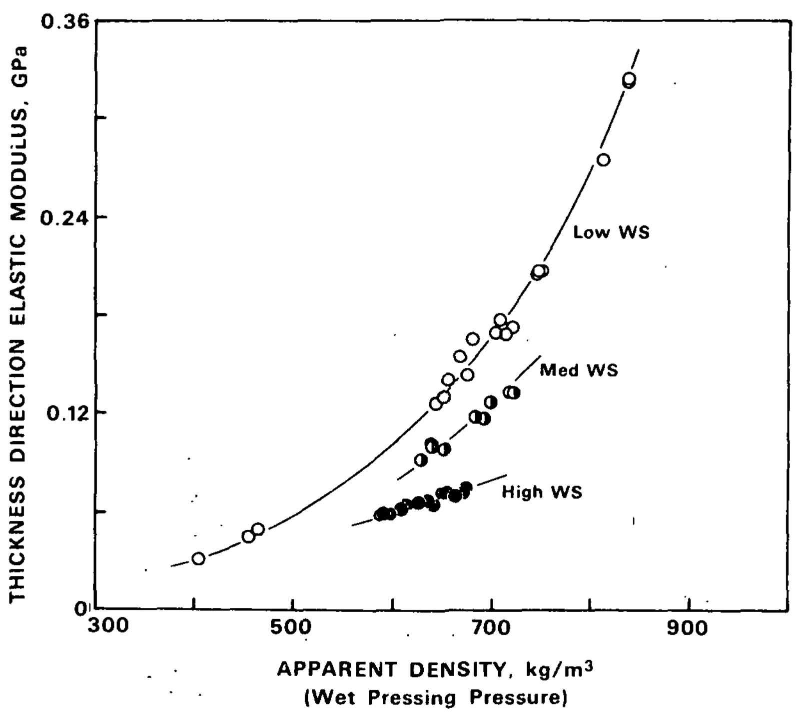

Figure 6. E_{zd} vs. density with changing fiber orientation and wet straining.

Of particular interest is the behavior of the out-of-plane Young's modulus E_z (~C_{33}). The in-plane fiber orientation has little impact on this out-of-plane parameter, but wet pressing and wet straining have very large influences. In Fig. 6 the magnitude of C_{33} increases by a factor of ten with wet pressing over the density range studied. On the other hand wet stretching degrades this property very markedly.

Examination of the results in Figs. 4-6 indicate that the effects of fiber alignment, wet pressing, and wet straining on the three moduli are not synergistic. The total effect is not greater than the sum of the independent effects.

Figure 7 shows the behavior of the three anisotropy ratios R_{xy}, R_{xz}, and R_{yz} as a function of wet straining. The anisotropy ratios are defined* as $R_{xz} = C_{11}/C_{33}$, $R_{yz} = C_{22}/C_{33}$, and $R_{xy} = C_{11}/C_{22}$. Two levels of wet pressing are shown. The in-plane anisotropy R_{xy} increases steadily with wet strain, as expected, since C_{11} is increasing (in the direction of wet straining) while C_{22} is decreasing. Above about 3.5% wet strain the sample ruptures. The small difference between the two levels of wet pressing for R_{xy} (solid and dashed lines) at zero wet strain most likely results from differences in the two sets of handsheets used, since wet pressing should not produce any in-plane anisotropy. At non-zero wet strains, however, it may be true that the higher wet pressing pressure leads to a different value for R_{xy}. (In these experiments the wet pressing operation was carried out before wet straining, just as on a paper machine.) The effect, however, appears to be small, at least in the range of wet pressure (densities) studied.

In the case of R_{xz} or R_{yz} at zero wet strain, the wet pressing pressure has quite a large effect on the out-of-plane anisotropy. Increasing the pressure from 25 psi (solid line) to 100 psi (dashed line) decreases R_{xz} (=R_{yz} at zero wet strain) from about 75 to 55. Higher pressing pressures probably would decrease this ratio more, although it is unlikely that the ratio would ever approach one, even with 100% bonding, because of the inherent anisotropy of the collapsed ribbon-like fibers.

Wet straining of the sample causes <u>both</u> R_{xz} and R_{yz} to increase. This happens even though C_{11} is increasing and C_{22} is decreasing (as in R_{xy}), because C_{33} is decreasing faster than C_{22}. It is apparent that a given level of R_{xz} (or R_{yz}) can be reached by different combinations of wet pressing and wet straining. The additional effects of refining and fiber orientation (in the plane) also need to be included in the analysis. The implications of these anisotropy ratios on end use performance needs to be established, of course.

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*The definitions use C_{ij} values defined in three dimensions. The values for C_{11} and C_{22} are actually those defined and measured in two dimensions. While the differences in values for the two definitions is essentially negligible, the distinction between them should not be forgotten.

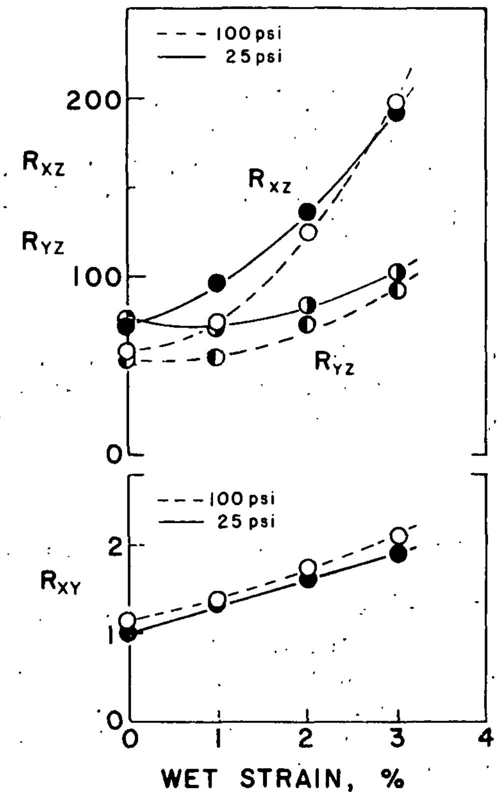

Figure 7. The three anisotropy ratios as a function of wet straining and wet pressing.

The effects of fiber orientation, wet pressing, and wet straining on the shear moduli are somewhat similar to those for the Young's moduli. Table II illustrates how the Young's moduli and shear moduli behave with increases in these three variables.

TABLE II

EFFECT OF MACHINE VARIABLES ON ELASTIC MODULI*

Modulus	Fiber Orientation (MD)	Wet Pressing	Wet Straining (MD)
MD	+	+	+
CD	-	+	-
ZD	0	+	- (lg.)
MD-CD Shear	-	+	0
CD-ZD Shear	-	+	-
MD-ZD Shear	+ (sm.)	+	- (sm.)

*+ increases
 - decreases
 0 no change

The effect of these variables on the Poisson ratios has not been extensively studied. While U_{xy} and U_{yx}, for example, are functions of wet straining and fiber orientation, their product is not very sensitive to these variables. The quantity $(U_{xy}U_{yx})^{1/2}$, is a measure of how interrelated the tensions in the MD are to those in the CD.

The results in Figs. 4-7 and Table II suggest that the elastic parameters for paper are not independent, but that process variables affecting a given parameter affect related properties in predictable ways. An example of this is an empirical relationship of the form $C_{66} = a(C_{11}C_{22})^{1/2}$ where a is a constant independent of machine variables if C_{11}/C_{22} is less than about 3.5 (18). Htun and Fellers (19) showed that the geometric mean of MD and CD properties are often invariant under the action of increased fiber orientation and wet stretching of the web. In the case of elastic parameters it seems that the geometric mean of the principal moduli in any plane is highly correlated with the shear moduli in that plane. Relationships similar to that above seem to exist in the other two symmetry planes as well, i.e. $C_{55} = b(C_{11}C_{33})^{1/2}$ and $C_{44} = c(C_{22}C_{33})^{1/2}$, where b and c are constants. Taken together, these suggest a single relationship between the the elastic parameters, viz.

$$C_{11}C_{22}C_{33} = K(C_{44}C_{55}C_{66})$$

Figure 8 shows the result of plotting all of the data for the samples shown in Figs. 4-7 in this manner. It does appear that the data fall along a straight line, with the intercept near zero. The interpretation of this observation is that any changes in paper machine variables will change the position along the line shown in Fig. 8, but that changes in the furnish (species, pulping, yield, or refining) would change the slope of the line. Recent work supports the idea that refining changes the slope of the line. Studies investigating the effects of yield are currently underway.

Fiber and Network Models

The discussion above treated paper as a homogeneous continuum. This is an acceptable viewpoint in many cases (e.g. the case of measurements using ultrasonic techniques), but paper, of course, is not a homogeneous continuum. It is a heterogeneous composition formed from fibers which themselves are heterogeneous. Figure 9 gives an idea of the complexity that must be considered in discussing a material such as paper or wocd.

Starting with "paper" on the lower left, we move upward to the "network". Here we are concerned with the arrangement of the fibers, the properties of the individual fibers, and the nature and frequency of bonds which occur between fibers. This area will be discussed in more detail later. A closer examination of the fibers reveals that the fibers are actually filament wound composite systems. The fibrils are arrangements of repeating cellobiose units or

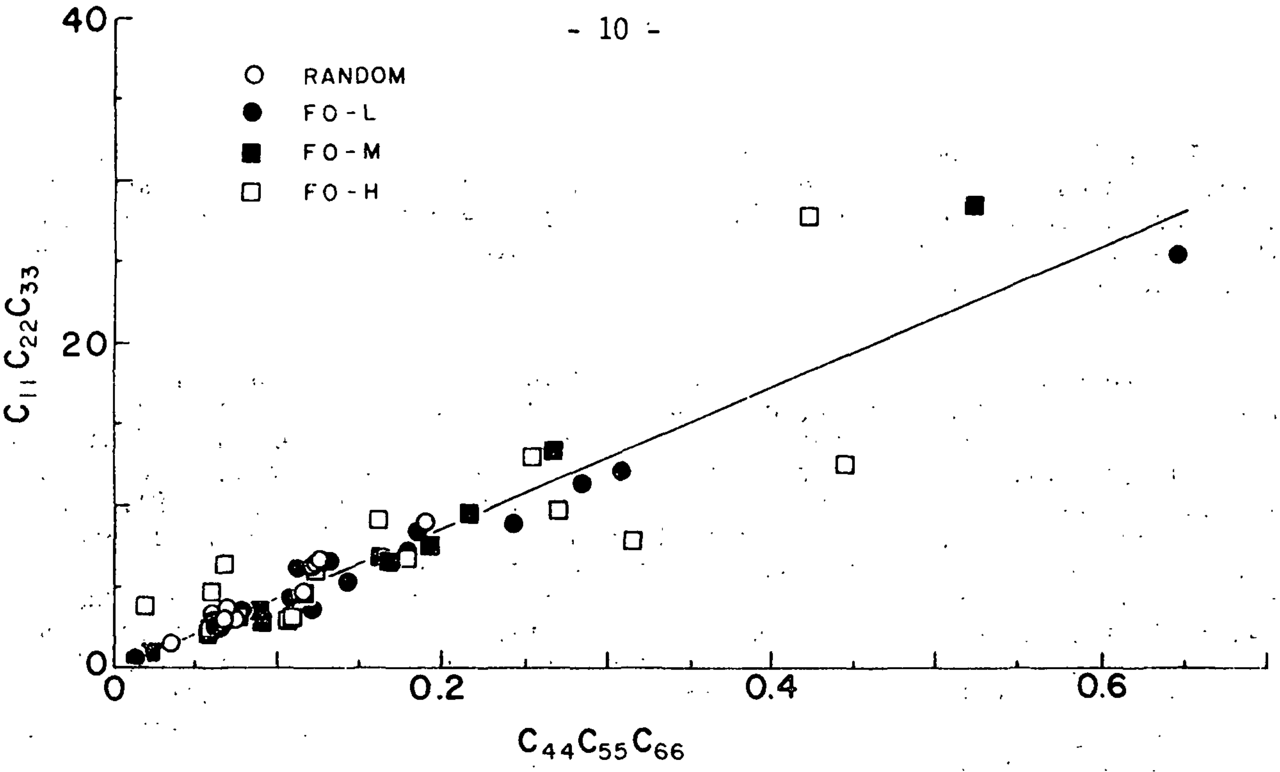

Figure 8. The product of the elastic axial stiffnesses vs. the product of the shear stiffnesses.

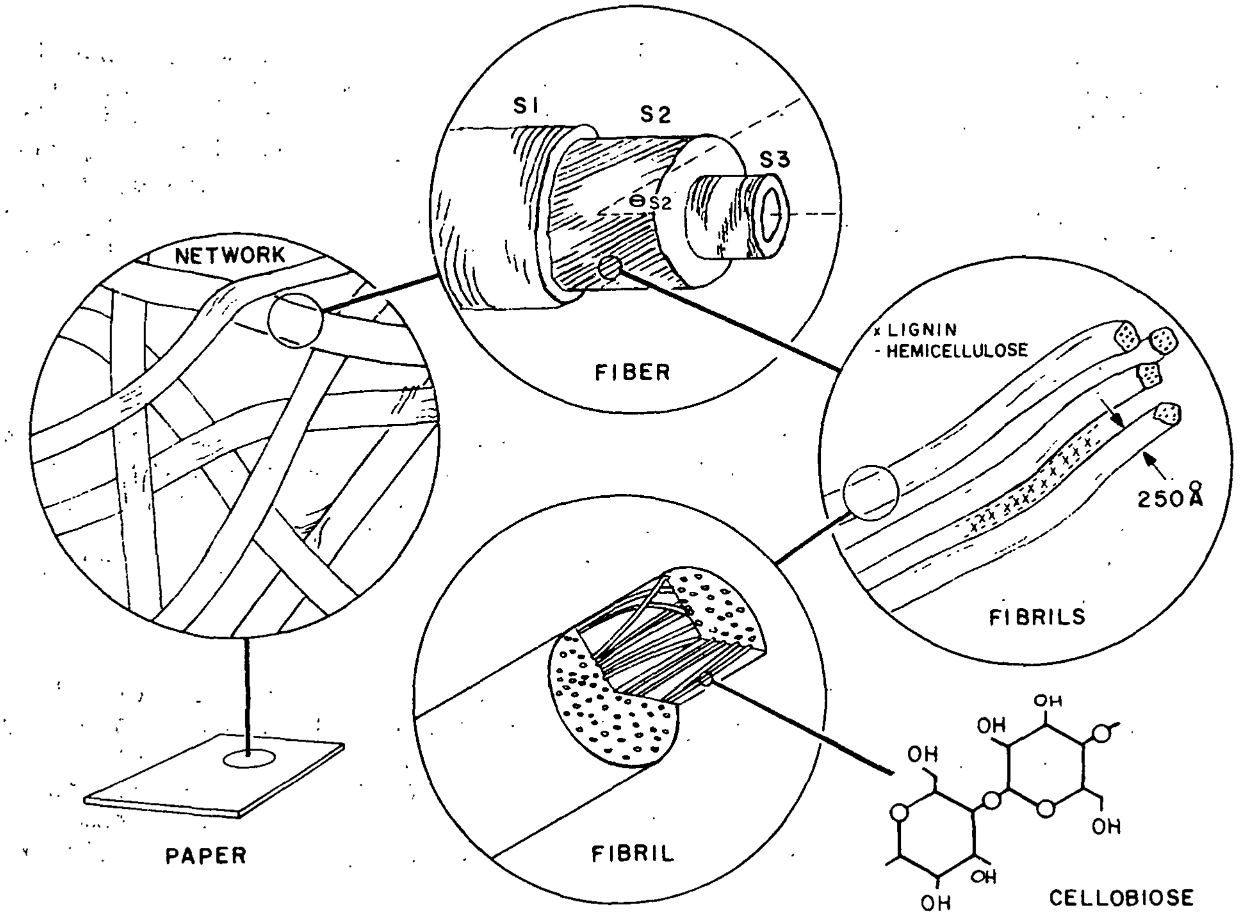

Figure 9. The heterogeneous nature of paper.

"chains". These chains may be arranged in an orderly way (crystalline) or non-orderly or random fashion (non-crystalline). The fibrils themselves are arranged in a regular fashion which differs in the various "layers" within the cell wall, and are held together by the hemicellulose and lignin matrix material. For a given wood species, fiber properties are clearly dependent upon the pulping process and yield, for these affect the integrity of the wood fiber. A low yield pulp, for example, in which the matrix materials have largely been removed, will be more flexible or "conformable" because the fibrils can easily move relative to one another when the fiber is swollen with water. Such a pulped fiber may also be more "beatable" because it is easier initially to get the water into the cell wall. In simple terms, the primary effects of beating are to cause this separation of the fibrils (or layers of fibrils) in the cell wall (internal fibrillation) and to cause partial unwinding of the outer layer(s) of the cell wall (external fibrillation). The latter can then also lead to the development of "fines". A negative aspect of refining is shortening of the fibers. All of the above effects are influenced by yield and pulping method and have been discussed extensively in the literature.

Two models of the fiber cell wall have been developed which relate the fibril and matrix properties, and the orientation of the fibrils relative to the fiber axis, to the overall fiber properties. Mark (20) developed a two-dimensional model including all of the layers of the cell wall, which can predict the axial and transverse fiber elastic properties as fibril and matrix parameters are varied and as the orientation of the fibrils relative to the fiber axis are varied. Page et al. (21) describe a three-dimensional model of the S2 layer (which constitutes about 70% of the cell wall) which is useful in examining the effect of the S2 fibril angle. The model is consistent with experimental data which show that fiber elastic stiffness and strength increase with decreasing fibril angle (measured between the fibril and the fiber longitudinal axis). A number of authors have shown that these same fiber properties are influenced by wet stretching or restraint during drying (22-24). It is believed (24) that these actions tend to remove defects in the cell wall or decrease the S2 fibril angle, both explanations requiring the cell wall to be swollen with water.

The change in fiber properties with drying has important consequences in terms of understanding the network properties. In addition to changes along the fiber axis, the swollen fiber also contracts laterally when it dries. Since fiber-to-fiber bonds have already formed prior to this shrinkage, the lateral contractions tend to compress the fibers to which they are bonded. The resultant "microcompressions" will affect the subsequent stress-strain behavior of the paper. Of course this phenomenon also explains why paper shrinks when it freely dries.

Network theories attempt to describe the elastic and tensile properties of the paper sheet in terms of individual fiber properties and the arrangement of fibers and fiber bonds. An accurate network model would enable one to predict sheet properties due to changes in furnish and machine variables. Most early theories considered paper as a homogeneous medium, neglecting the fibrous nature. More recently, linear and nonlinear network theories have been developed.

Real fiber networks do not easily lend themselves to mathematical modeling since one cannot talk about a single fiber modulus, diameter, length, or stiffness, because all of these parameters must be expressed as statistical distributions. Furthermore, because the fiber mechanical properties may change locally during consolidation and especially drying of the paper web, any model which assumes constant fiber properties will come up short in describing actual sheet properties. The fiber properties determined for single fibers are not apt to be the properties the same fiber would have had it been dried in the paper web. Nevertheless such models are useful in understanding how properties develop in the sheet and what might be done to improve sheet properties.

Most network theories to date have dealt with two-dimensional models and have involved many simplifying assumptions. Modes of response to stress which might be important are often neglected. The loads applied to the fibers in a network act at numerous localized areas on the fiber surfaces. This means the stresses and strains within the fibers must be highly variable. The nature of the bonding and the microcompressions referred to above also add to this variability. Most early models did not consider the anisotropic nature of paper.

Cox (25) developed the original theory for a random fiber network consisting of long straight fibers attached at the ends. He assumed that the strain in the fibers was equal to the strain in the sheet. This type of theory is referred to as a uniform strain theory. Cox found that for a random network, $E_s = 1/3E_f$, where E_s is the sheet modulus and E_f is the fiber modulus.

Several early theories made faulty assumptions which led to incorrect results. Space does not allow a detailed description of any one of the existing models because of their complexity, but several of the better known are listed in Table III. The equations in Table III are not necessarily as published but have been reduced, where possible, to a comparable form. All of the equations assume planar mats, mean values for fiber properties, and random fiber distributions.

The most recent models of Perkins et al. (30) and Page et al. (32) have led to a better understanding of the dependence of sheet modulus on fiber modulus. Page and Seth have also shown that the entire stress-strain response for paper can, in fact, be related to the stress-strain response for the fibers themselves. These latter theories are still being developed and evaluated. Time will judge how successful they will be.

TABLE III

SOME NETWORK THEORIES

Author	Reference	Sheet Modulus
Cox (1952)	25	$\frac{1}{3}\frac{D}{d}E$
Onogi and Sasaguri (1961)	26	$\frac{8}{\pi^2}\left[\frac{1}{g^2/3r^2+1}\right]\frac{D}{d}E$
Campbell (1963)	27	$\frac{1}{3}\frac{D}{d}E$
Van den Akker (1962)	28	$\frac{1}{3}\left[1+\frac{41G}{aGb^2+12EI+2GI}\right]\frac{D}{d}E$
Kallmes (1963)	29	$\frac{1}{3}\left[1+\frac{16I}{3aGb^2+36EI+8GI}\right]\frac{D}{d\tau}E$
Perkins & Mark (1976)	30	$\frac{1+2\beta}{3+2\beta}\left[\frac{1}{1+\frac{3}{2}\left(\frac{2a_o}{tf}\right)^2}\right]\frac{D}{d}E$
Kallmes (1977)	31	$\frac{1}{3}(1-f_i)\frac{D}{d}E$
Page & Seth (1979)	32	$\frac{1}{3}\left[1-\frac{w}{gRBA}\sqrt{\frac{E}{2G}}\right]\frac{D}{d}E$

a = cross-sectional area of fiber wall
d = density of fiber wall
D = sheet density
E = fiber modulus
g = fiber segment length
G = fiber shear modulus
I = moment of inertia of fiber cross section
β = dimensionless parameter which is a function of fiber geometry and elastic properties and a_o

b = unbonded fiber segment length
r = fiber radius for circular fibers
RBA = relative bonded area
w = fiber width for rectangular fibers
τ = fiber curl
t_f = fiber thickness for rectangular fibers
a_o = measure of slackness in unstrained network
f_i = initial fraction of inactive fibers

More recently composite models have been applied to paper. These treat paper as having a homogeneous structure (33,34). A good deal of insight can be gotten from such models. The most recent of these uses the cell-wall layer of the fiber as the primary building unit such that the macroscopic properties of paper are related to the properties of the constituent polymers, hemicelluloses, and cellulose (34).

Viscoelastic Nature of Paper

An elastic material is one which changes its shape and size under the action of applied stresses, but recovers its original configuration immediately when the stresses are removed. Implicit in this definition is the requirement that the stresses in Equation 1 follow the strains instantaneously. If this is not the case, and the stress-strain relationships are time dependent, the material is said to be viscoelastic. Paper fits this description since it exhibits creep (elongation over time at constant load) and stress relaxation (decrease of stress with time at constant deformation). In cyclic stressing, the resultant strain will not be in phase with the applied stress. All of these can affect the usefulness of paper in end use applications. Kolseth and de Ruvo (35) have recently reviewed the viscoelastic response in paper and discussed methods for measuring it. The interested reader is referred to that work since it is beyond the scope of this review to cover the subject in any detail. Of prime importance here is whether the (time independent) elastic parameters will relate to end use performance at all. In the next section we see that this indeed is the case.

Figure 10 shows an idealized frequency spectrum for the storage modulus (viscoelastic modulus) E' and loss tangent for a "dry" and "moist" sample of cellophane (36). E' is sensitive to measuring frequency temperature, and moisture content (37,38). The molecular motions responsible for the "γ" and "β" relaxations are not certain, but they do behave differently with increasing moisture content. Results such as those in Fig. 10 are helping us to understand better the complex relationships between time, temperature and moisture in cellulosic materials.

Elastic Parameters and End Use Performance

Most paper specifications involve tests which are taken to be descriptive of the end-use performance of the material. Such tests are usually destructive. It is surprising then, that many of the common paper tests correlate with certain elastic parameters, over broad ranges of values. This is of significance because it is possible to measure the three-dimensional elastic properties of most papers nondestructively using ultrasonic methods. Measurements made on a single specimen can be used to predict a number of destructive test values. In this way it is quite easy to study the effect of process variables on end-use tests as well as monitor product quality. Later we will mention how such measurements can also be made on the paper machine to indicate product quality on a continuous basis and to optimize machine operating parameters. At some time in the future it may be possible to control the paper machine using such on-machine measurements of elastic properties.

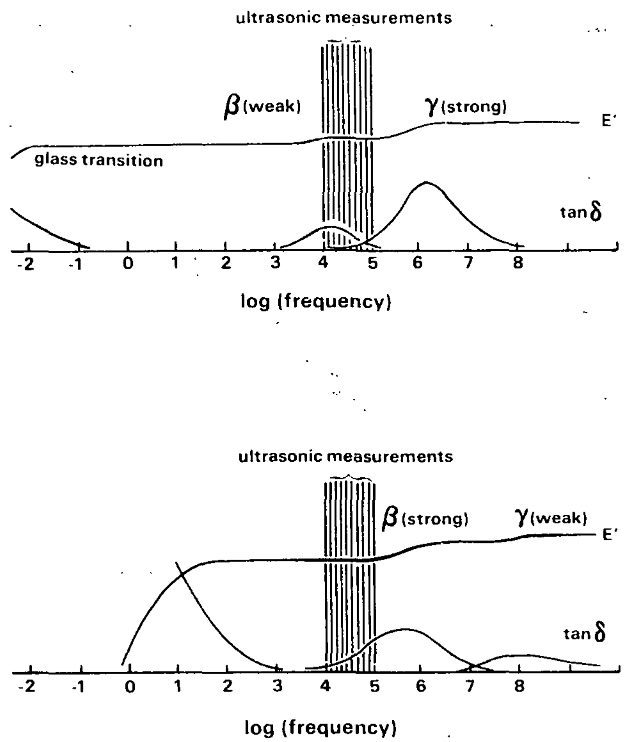

Figure 10. Idealized frequency spectrum of the real part of the complex modulus and the loss tangent for dry (top) and 7% moisture content (bottom) cellophane (36).

Figure 11 shows how MD or CD tensile strength varies with E_{md} or E_{cd}. All of the data, covering a rather broad range of tensile strengths (either MD or CD) fall along a single line. The tensile strengths were varied by changing fiber orientation, wet pressing, wet straining and drying restraints. Figure 12 shows how the ZD tensile strength (sometimes used as a measure of internal bond strength) varies with C_{33} (or E_{zd}) for the same array of samples. The last two figures suggest that a given elastic modulus might be used to predict tensile strength or to monitor the changes in MD, CD and ZD tensile strength with process changes. Measurements on only one specimen would be required to do this. Similar results to those of Figs. 11 and 12 are obtained if one compares density specific parameters, i.e. breaking length versus E/ρ. Such correlations have also been shown to hold for machine made papers.

Figure 13 shows MD and CD STFI compressive strengths plotted against the products of in-plane and out-of-plane elastic parameters. In this case a theory does relate the compressive failure with the product of elastic parameters (39). The theory predicts that CD compressive strength, for example, should correlate with $(E_{cd} \cdot G_{cd-zd})^{\frac{1}{2}}$. The experimental data give a best fit line with a slope of 0.49, in good agreement with theory.

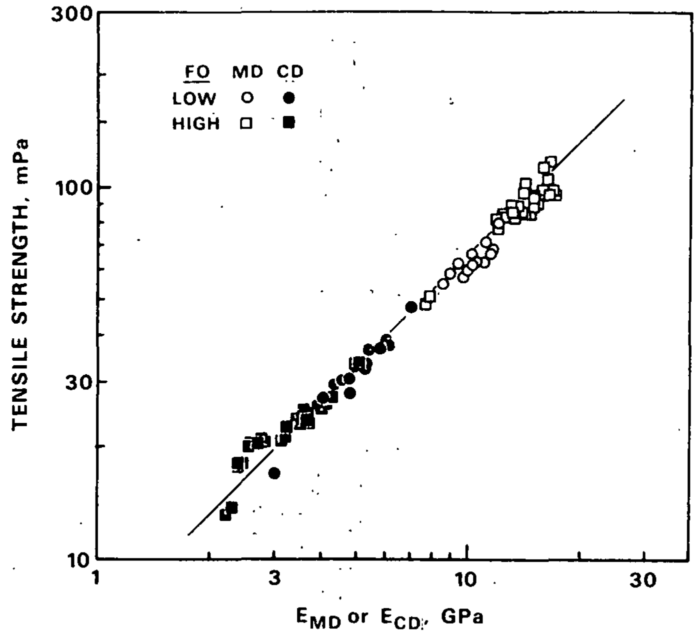

Figure 11. MD and CD tensile strengths plotted against E_{md} and E_{cd}, respectively.

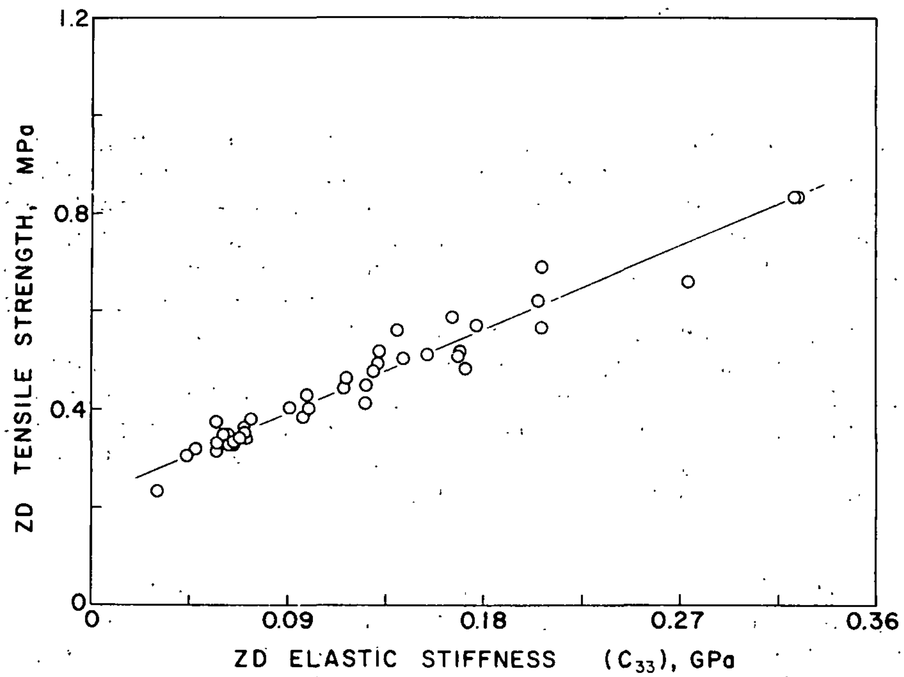

Figure 12. ZD tensile strength plotted against the ZD elastic stiffness, C_{33}.

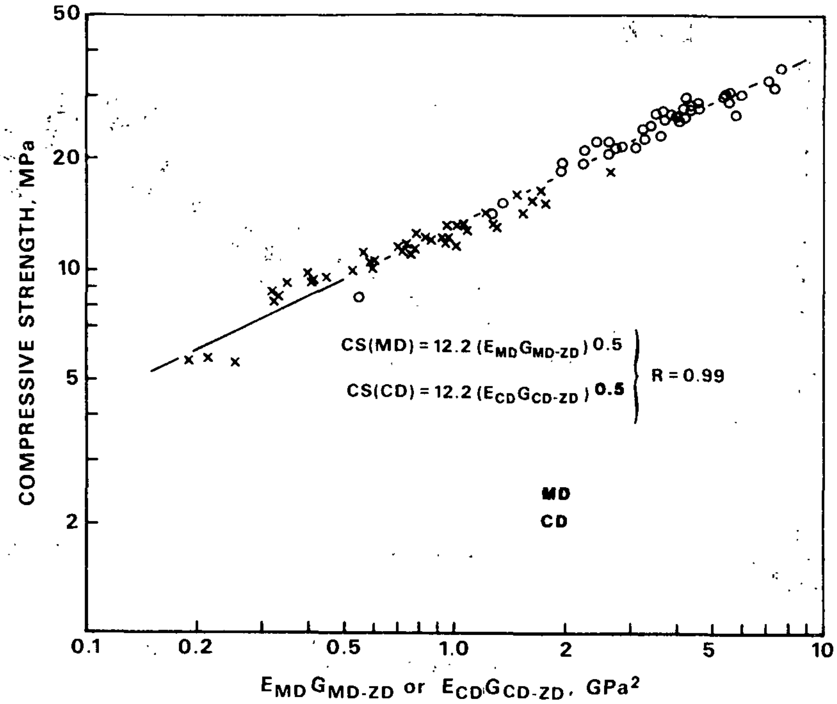

Figure 13. MD and CD STFI compressive strength plotted against the products $E_{md} \cdot G_{md-zd}$ and $E_{cd} \cdot G_{cd-zd}$, respectively. The regression line coincides with the expected behavior (37).

Figure 14 shows specific bursting strength plotted against the sum $(E_{md}+E_{cd})/\rho$. It is generally taken that bursting strength is proportional to the average tensile strength, thus from Fig. 11 it follows that the behavior of Fig. 14 is to be expected. Dividing the bursting strength by density has removed the strong basis weight dependence and collapsed the data (four grades of linerboard) onto a single line.

Table IV lists the above and other relationships that have been discovered to date between certain commonly measured physical properties of paper and the elastic parameters (40).

While it is true that the relationships described may not be valid for all paper grades or basis weights, the use of elastic parameters to evaluate end-use performance and to study the interactions between process variables and paper properties has so far been very productive.

A practical application of the correlations between elastic parameters and paper quality factors has been made in a device which measures paper properties on the paper machine. This sensor, installed on a linerboard machine in the

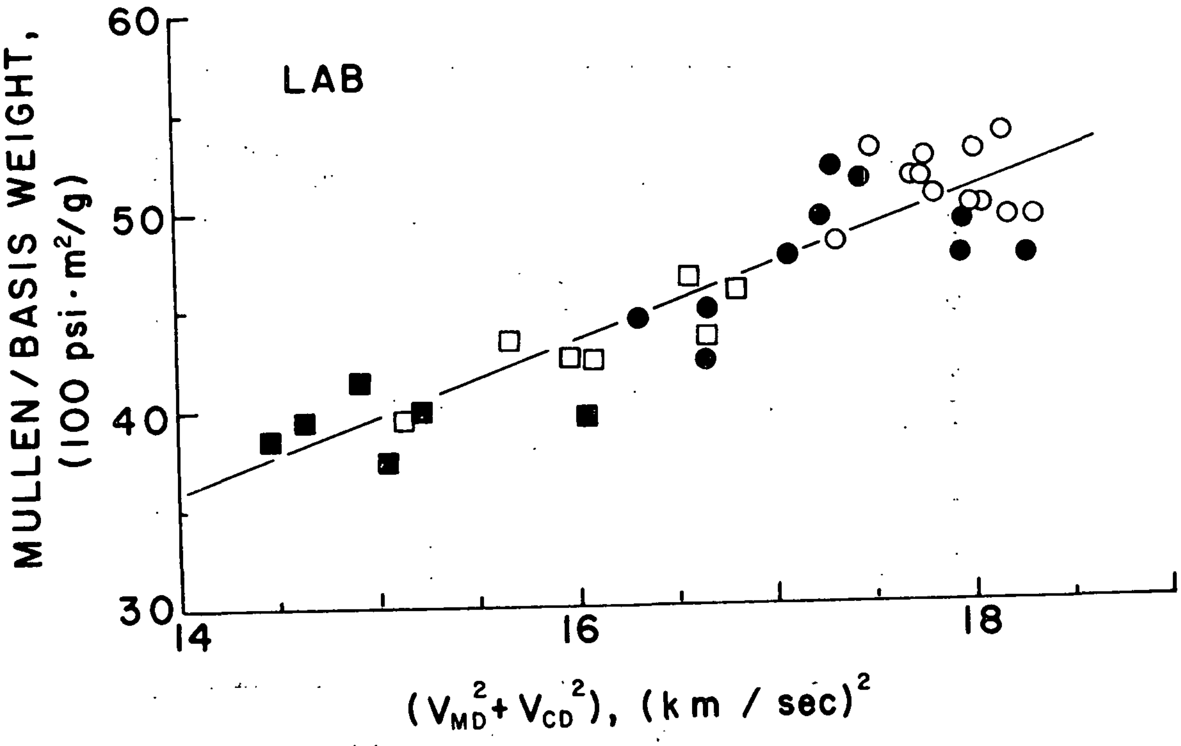

Figure 14. Specific bursting strength vs. the sum of E_{md}/ρ and E_{cd}/ρ.

TABLE IV

END USE TESTS AND ELASTIC PARAMETERS

Property	Elastic Parameters
MD Tensile Strength	E_{md}
CD Tensile Strength	E_{cd}
ZD Tensile Strength	E_{zd}
MD/CD Tensile Ratio	E_{md}/E_{cd}
MD Compressive Strength	$E_{md} \cdot G_{md-zd}$
CD Compressive Strength	$E_{cd} \cdot G_{cd-zd}$
MD Bending Stiffness	$E_{md} \cdot \text{Caliper}^3$
CD Bending Stiffness	$E_{cd} \cdot \text{Caliper}^3$
Internal Bond Strength	E_{zd}
Bursting Strength	$E_{md}+E_{cd}$
Flutability	E_{md} and G_{md-zd}
Combined Board Performance	E_{cd} and G_{cd-zd}

southern United States since January, 1983, measures E_{md} and G_{md-cd} of the moving paper web (41). After correcting the values for moisture and temperature variations, they are used to predict the bursting strength, CD ring crush, and CD-STFI compressive strength of the linerboard on a continuous basis.

The real payoff for such a sensor, however, is probably not in product quality measurements, but in paper machine control. Both E_{md} and G_{md-cd} are sensitive to process and paper machine variables (but in different ways) and thus permit "tuning" the paper machine to provide optimum board properties. Eventually this capability could lead to automatic control of the papermaking process.

In summary, the elastic properties of paper seem to form a basic set of parameters which are useful for monitoring the effects due to changes in process variables, capable of predicting end use performance, and, overall, helping us to better understand the fibrous network we call paper. Elastic parameters are also important, of course, in product design and modeling, e.g. in the construction of tubes, boxes, food containers, etc. Because most of the elastic parameters needed to describe paper can now be determined easily and nondestructively using wave propagation methods, the opportunity exists to move forward in each of these areas.

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